Section 10.2

Banking Applications

Basic facts

\[ P = \text{principal or the amount of money that we want to invest now} \]
\[ t = \text{time in YEARS. If the time is given in months then you must divide by 12. For example 6 months} = \frac{6}{12} \text{ years}. \]
\[ r = \text{annual interest rate as a decimal. If the rate was 3% then} \quad r = 0.03 \]
\[ n = \text{number of compounding periods or how often the interest is calculated and added back to the principal.} \]
\[ A = \text{amount of money that we will have after the principal has stayed in the bank for} \ t \ \text{years, with an annual rate of} \ r \ \text{and with number of compounding periods} \ n \]
\[
\begin{align*}
 n &= 1 \quad \text{annually} \\
 n &= 2 \quad \text{biannually} \\
 n &= 4 \quad \text{quarterly} \\
 n &= 12 \quad \text{monthly} \\
 n &= 52 \quad \text{weekly} \\
 n &= 365 \quad \text{daily}
\end{align*}
\]

Fixed number of Compounding Periods

The first formula depends on a fixed number of compounding periods:

\[ A = P \cdot \left(1 + \frac{r}{n}\right)^{nt} \]

Example

money now is $2000, \quad P = 2000 \\
annual rate is 3\%, \quad r = 0.03 \\
compounding quarterly, \quad n = 4 \\
time in the bank is 5 years, \quad t = 5 \]
Continuos Compounding

In this situation the interest is recalculated and re-added to the principal all of the time. You can think of it as letting the number of compounding periods get larger and larger.

Our old formula needs some modification and in doing so we need a new exponential function called "e". "e" is a famous number like pi and it does have a value

\[ e = 2.7182... \]

We can even graph

\[ f(x) = e^x \]

The graph in red is our friend \( e^x \), the graph in green is \( 2^x \) and the graph in blue is \( 3^x \). They all behave very similarly.

Now that we have a grasp of "e" our for continuos compounding which means that
the compounding is being done all of the time is below:

\[ A = P \cdot e^{rt} \]

If you have a hard time finding "e" on your calculator it is above the "ln" key.

Using the numbers that we had above

\[ A = 2000 \cdot e^{0.03 \cdot 5} \]
\[ A = 2000 \cdot e^{0.15} \]
\[ A = 2323.70 \]

It is not a lot more that our Fixed Compounding but it the best that is possible.