Section 2.5

Basics

Inequalities come in four flavors

\[ \leq, \geq, >, < \]

The ones with the lines under then include the possibility of equality

\[ 2 \leq 2 \text{ is true} \]
\[ 2 < 2 \text{ is false} \]

I tend to think of them as the "pointy side" points to the smaller item. If you keep that thought in mind the following are the same

\[ 2 < 3 \text{ or } 3 > 2 \]
\[ 1 < x < 6 \text{ or } 6 > x > 1 \]

Notice that all of the following are true which says that similar to equations as long as we do the same thing to both sides the inequality remains the same.

\[ 2 + 1 < 3 + 1 \]
\[ 2 - 1 < 3 - 1 \]
\[ 2 \times 3 < 3 \times 3 \]
\[ \frac{2}{4} < \frac{3}{4} \]

There is one exception though which is multiplying or dividing both sides by a negative number.

\[ 2 < 3 \]
\[ 2 \times -2 < 3 \times -2 \text{ is false} \]
\[ 2 \times -2 > 3 \times -2 \text{ is true} \]

Interval Notation

This notation is a way to represent a collection of numbers on the number line. The format is show below

\[ ( \text{ or } [ \text{ smaller item, larger item } ] \text{ or } ] \]

Some symbols
off to the right forever $\infty$
off to the left forever $-\infty$

( or ] and when to use

( or ) go with $>$ or $<$ and with $\pm \infty$
[ or ] go with $\geq$ or $\leq$

Examples

\begin{align*}
x > 3 \text{ is } (3, \infty) \\
x \leq 3 \text{ is } (-\infty, 3] \\
1 < x \leq 3 \text{ is } (1, 3] \\
\text{all real numbers is } (-\infty, \infty)
\end{align*}

Later in the course we will see intervals with "split" pieces such as the following
\[ x < 3 \text{ or } x > 5 \text{ which is } (-\infty, 3) \cup (5, \infty) \]

The $\cup$ stands for union. If you see a $\cap$ this stands for intersection or the part in common.

Danger

Finally be very careful as in the next example.
\[ 2 > x \]

Some students try to tell me that because the item points to the right the collection is everything bigger than 2. Actually if you "try" 3 you will see this cannot be the case.

The notation is really
\[ (-\infty, 2) \]

Example #20, page 80

\begin{align*}
-3x &\leq 9 \\
x &\geq \frac{9}{-3} = -3 \\
x &\geq -3 \\
&\quad \quad [\,-3, \infty) \n\end{align*}

Example #54, page 80
\[ 3(x - 1) \geq -(x + 4) \]
\[ 3x - 3 \geq -x - 4 \]
\[ 4x - 3 \geq -4 \]
\[ 4x \geq -1 \]
\[ x \geq \frac{-1}{4} \]
\[ \left[ \frac{-1}{4}, \infty \right) \]

Example #70, page 80

\[ -2(x - 4) < 5(x - 1) \]
\[ -2x + 8 < 5x - 5 \]
\[ 8 < 7x - 5 \]
\[ 13 < 7x \]
\[ \frac{13}{7} < x \]
\[ \left( \frac{13}{7}, \infty \right) \]

Example #74, page 81 Strange Cases

\[ 5x - 2 > 5x + 3 \]
\[ -2 > 3 \]

No solution since this is never true.

\[ 3x - 4 < 3x - 7 \]
\[ -4 < 7 \]

Where did the variables go?

This is always true so

\[ (-\infty, \infty) \]