More Fractional Equations

Fractions have a top part called the numerator and a bottom part called the denominator. All fractions are of the form

\[ \frac{p}{q} \]

where p and q any numbers you like, EXCEPT q cannot be ZERO

Example

Solve the following equation

\[ \frac{5y - 4}{6y^2 + y - 12} - \frac{2}{2y + 3} = \frac{5}{3y - 4} \]

First we factor all of the terms.

Our first try on factoring should be the terms that we already have there

\( (2y + 3)(3y - 4) \)

and yes

\( (2y + 3)(3y - 4) = 6y^2 + y - 12 \)

\[ \frac{5y - 4}{(2y + 3)(3y - 4)} - \frac{2}{2y + 3} = \frac{5}{3y - 4} \]

and the bad answers ( make a denominator = 0 ) are

\[ 2y + 3 = 0 \]
\[ 2y = -3 \]
\[ y = -3/2 \]
\[ 3y - 4 = 0 \]
\[ 3y = 4 \]
\[ y = 4/3 \]

Now multiply all terms by

\( (2y + 3)(3y - 4) \)

to get
\[
(2y + 3)(3y - 4) \left[ \frac{5y - 4}{(2y + 3)(3y - 4)} - \frac{2}{2y + 3} \right] = (2y + 3)(3y - 4) \left[ \frac{-5}{3y - 4} \right]
\]
\[
5y - 4 - 2(3y - 4) = 5(2y + 3)
\]
\[
5y - 4 - 6y + 8 = 10y + 15
\]
\[
-y + 4 = 10y + 15
\]
\[
-11 = 11y
\]
\[
-1 = y
\]

and since this is not one of the bad answers we keep it.

**More Word Problems**

**Example**

An inlet pipe can fill a tank in 12 minutes. A drain can empty the tank in 18 minutes. If the tank is empty, and both the pipe and drain are open, how long will it take before the tank overflows?

Suppose the tank holds \( V \) gallons.

If our inlet flows at \( x \) gallons per min for 12 min we know it will fill the tank.

\[
x \cdot 12 = 12x = V
\]
\[
x = \frac{V}{12}
\]

We also know that there is an outlet which flows out at \( y \) gallons per min and in 18 min the tank will empty.

\[
y \cdot 18 = 18y = V
\]
\[
y = \frac{V}{18}
\]

The tricky part.

If the tank will overflow then the **rate in** \( x \) must be faster than the **rate out** \( y \) and so \( x - y \) is the **net flow inward**.

\( T \) is the time it will take we reach \( V \) gallons.
\[(x - y)T = V\]
\[
\left( \frac{V}{12} - \frac{V}{18} \right)T = V
\]
\[
\left( \frac{3V}{36} - \frac{2V}{36} \right)T = V
\]
\[
\frac{V}{36}T = V
\]
\[
T = 36 \text{ min}
\]