Section 6.2

Quadratic Equations

First off, what does this equation thing look like?

\[ ax^2 + bx + c = 0 \]

The \( x \) denotes that variable in the equation and while in this case it is labeled \( x \) it could be other letters as well.

The \( a, b, c \) are fixed real numbers and while in this case there is a \(+\) before the \( b \) and the \( c \) it could just as well be a \( - \).

These three numbers can be any number that you like with only 1 exception and that is \( a \) CANNOT be ZERO.

Examples

\[ 5x^2 - 11x + 4 = 0, \quad a = 5, \quad b = -11, \quad c = 4 \]
\[ 3x^2 - 14x = 0, \quad a = 3, \quad b = -14, \quad c = 0 \]
\[ x^2 - 4 = 0, \quad a = 1, \quad b = 0, \quad c = -4 \]
\[ 2x^2 = 0, \quad a = 2, \quad b = 0, \quad c = 0 \]

Where did these equations come from?

\[ f(x) = ax^2 + bx + c \]

They came from a type of functions called quadratic functions which is part of a larger class called polynomial functions of degree \( n \).

\[ f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n \]

Quadratic functions look like a \( U \) that looks up \((a > 0)\) or looks down \((a < 0)\).

Example

\[ f(x) = x^2 - 4 \]
Example

\[ f(x) = -x^2 - 1 \]

The reason that I brought this up is because our quadratic equation

\[ ax^2 + bx + c = 0 \]

answers the question: Where does the quadratic function (parabola) cross on the \( x \) axis or said a different way where are the \( x \) intercepts?

There can be only 3 answers:

- Crosses in 2 places, so 2 real number roots or solutions
- Crosses in 1 place, so 2 real numbers roots that are repeated
- Never crosses or touches \( x \) axis, so 2 complex number solutions

We do not need to recall a lot of information about complex numbers but the following basic information is important.

\[ \sqrt{-4} = \sqrt{4} \times -1 = \sqrt{4} \times 1 = 2i \]

**Solving Quadratic Equations**

In this section and the next we will see four ways to solve these equations.
1. By Factoring \(\Rightarrow\) Easy but then again not many items factor

2. By Square Root Property \(\Rightarrow\) Requires a special format and is based on factoring

3. By Completing the Square \(\Rightarrow\) Long Process and simply converts quadratic equation into Square Root Property.

4. By the Quadratic Formula \(\Rightarrow\) Handy thing, always works, never leave home without it.

**Zero Factor Property**

Keep reminding yourself that factoring only works if you have 0 on one side and everything else on the other.

**Example**

Solve the following quadratic equation by **factoring**.

\[
x^2 - 7x + 10 = 0
\]

Solution: Factor it as

\[
x^2 - 7x + 10 = 0 \\
(x - 2)(x - 5) = 0
\]

Then set each part equal to zero

\[
x - 2 = 0 \text{ and } x - 5 = 0 \\
x = 2 \text{ and } x = 5
\]

**Example**

\[
x^2 - 49 = 0 \\
(x - 7)(x + 7) = 0 \\
x = -7, +7
\]

**Square Root Property**

This property says that if you can get your problem in the format

\[
(\text{blob of symbols})^2 = \text{number}
\]

then you can conclude by factoring

\[
\text{blob of symbols} = \pm \sqrt{\text{number}}
\]
Note
\[ \sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2} \]
\[ \sqrt{72} = 8.485 \]

I am not picky about this and you can stop at square root of 72. Normally to get your answer you just use your calculator.

Example

Solve the following quadratic equation by the square root property.

\[ (2x - 1)^2 = 23 \]

Solution:

\[ 2x - 1 = \pm \sqrt{23} \]
\[ 2x = 1 \pm \sqrt{23} \]
\[ x = \frac{1 \pm \sqrt{23}}{2} \]

Example

\[ (x - 3)^2 = -16 \]
\[ x - 3 = \pm \sqrt{-16} \]
\[ x = 3 \pm 4i \]