Section 6.4

Quadratic Equations - Quadratic Formula

The formula is derived from Completing the Square.

\[ ans = \frac{-1 \pm \sqrt{b^2 - (4 \times a \times c)}}{2 \times a} \]

There are only a few things to comment on the formula:

1. Your problem must be in the standard format before you can read off \(a\), \(b\), and \(c\)

   \[ ax^2 + bx + c = 0 \]

   \[ x^2 - 16 = 0, \quad a = 1, b = 0, c = -16 \]
   \[ 4x^2 - 3x + 11 = 0, \quad a = 4, b = -3, c = 11 \]

2. \(-b\) really means \(-1 \times\) whatever \(b\) happens to be

3. \(4ac\) can turn out to be negative so make sure you do not lose track if you have a double negative.

4. \(b^2 - (4ac)\) is called the discriminant and its value positive, negative or zero determines the nature of your answers.

Example

Solve the following quadratic equation by the **quadratic formula** and get decimal answers.

\[ 3.2x^2 - 4.1x - 10.4 = 0 \]

Solution:

\[ a = 3.2 \text{ and } b = -4.1 \text{ and } c = -10.4 \]

Plug into the formula
\[
\text{ans} = \frac{-1 \times -4.1 \pm \sqrt{(-4.1)^2 - (4 \times 3.2 \times -10.4)}}{2 \times 3.2}
\]
\[
\text{ans} = \frac{4.1 \pm \sqrt{(-4.1)^2 - (4 \times 3.2 \times 10.4)}}{6.4}
\]
\[
\text{ans} = \frac{4.1 \pm \sqrt{16.81 - (-133.12)}}{6.4}
\]
\[
\text{ans} = \frac{4.1 \pm \sqrt{149.93}}{6.4}
\]
\[
\text{ans} = \frac{4.1 \pm 12.24}{6.4}
\]
\[
\text{ans} = 2.55, \ -1.27
\]

**Discriminant**

\[b^2 - (4 \times a \times c) > 0\] means that there are 2 different real answers

\[b^2 - (4 \times a \times c) < 0\] means that there are 2 different complex answers

\[b^2 - (4 \times a \times c) = 0\] means that there is 1 real answer